To prove that a heuristic is consistent, it is also admissible, there are two important properties to look at:

**Admissibility**: A heuristic is admissible if it never overestimates the actual cost of reaching the goal from any given state.

**Consistency** (or the triangle inequality): A heuristic is consistent if the estimated cost from a state to its successor, plus the estimated cost from the successor to the goal, is never less than the estimated cost from the current state to the goal.

Let's proceed with the proof:

**Proof**: If a heuristic is consistent, it is also admissible.

An example we can show is if we have a consistent heuristic h(n) for a given state n.

Admissibility:

We need to show that the consistent heuristic h(n) is admissible, like we stated above, it means that it never overestimates the actual cost of reaching the goal from any state.

By contradiction, suppose h(n) overestimates the actual cost from a state n to the goal. Let g\*(n) represent the actual cost of reaching the goal from state n.

Since h(n) is consistent, according to the triangle inequality, we have:

h(n) ≤ cost(n, succ) + h(succ) for any successor state succ of n.

Let's consider a successor state succ with the minimum cost:

g\*(n) = cost(n, succ) + g\*(succ) (1)

Since h(n) overestimates g\*(n), we have:

h(n) > g\*(n)

Substituting equation (1) in the inequality:

h(n) > cost(n, succ) + g\*(succ)

Rearranging the terms:

h(n) - g\*(n) > g\*(succ) (2)

Now, we consider the consistent heuristic h(succ) for the successor state succ. According to the consistency property:

h(succ) ≤ cost(succ, goal) + h(goal)

Since succ is a successor of n, we can rewrite the inequality as:

h(succ) ≤ cost(n, succ) + h(goal)

Adding g\*(succ) to both sides:

h(succ) + g\*(succ) ≤ cost(n, succ) + g\*(succ) + h(goal)

Using equation (2), we can substitute:

h(succ) + g\*(succ) ≤ h(n) - g\*(n) + h(goal)

h(succ) + g\*(succ) + g\*(n) ≤ h(n) + h(goal)

Since h(goal) is a constant, we can ignore it:

h(succ) + g\*(succ) + g\*(n) ≤ h(n)

This implies that h(succ) + g\*(succ) ≤ h(n) - g\*(n).

But this contradicts the assumption that h(n) is consistent, as it violates the triangle inequality.

Hence, our assumption was incorrect, and h(n) does not overestimate the actual cost of reaching the goal from state n. Therefore, the consistent heuristic h(n) is admissible.

Through our example, we have proved that if a heuristic is consistent, it is also admissible.

**Now, let's provide an example of a heuristic that is admissible but not consistent:**

Example:

Consider a heuristic h(n) that estimates the number of misplaced tiles in a sliding puzzle game. In this game, the goal state is to have all tiles in their correct positions.

Admissibility:

The heuristic h(n) is admissible because it never overestimates the actual cost. For the best case, it accurately estimates the number of misplaced tiles, and for the worst case, it underestimates the actual cost by ignoring the cost of moving tiles.

Consistency:

The heuristic h(n) is not consistent. For example, let's consider two adjacent states in the sliding puzzle game, where one tile needs to move from position A to position B. In one move, the tile can move from A to B. Therefore, the cost from A to B is 1. However, the heuristic value h(A) might be 2 (overestimation) if there are two misplaced tiles, and h(B) might be 0 (underestimation) if B is a correct position for the tile. Hence, h(A) > 1 + h(B), violating the consistency property.

Therefore, the heuristic h(n) that estimates the number of misplaced tiles in the sliding puzzle game is admissible but not consistent.

It's important to mention that admissibility and consistency are desirable properties for heuristics used in informed search algorithms. Admissible heuristics guarantee optimality, while consistency helps in improving the efficiency of search algorithms.